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Performance Comparison of MUSIC and ROOT-MUSIC Algorithm for Estimation of DOA in Uniform Linear Array

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Abstract—*The smart antenna technology depends on antenna arrays where the radiation pattern is modified by altering the amplitude and relative phase of the different elements. The capability of smart antenna greatly relies upon the viability of direction of arrival (DOA) estimation algorithm. The DOA algorithms estimate the number of electromagnetic waves encroaches on the antenna array and their angle of arrival. This paper differentiates the execution of the multiple signal classification (MUSIC) and ROOT-MUSIC algorithm on the uniform linear array (ULA). From the simulation results, it is observed that ROOT-MUSIC algorithm indicate accurate estimation than the MUSIC algorithm. Further, we have demonstrated that ROOT-MUSIC provides higher direction finding accuracy and higher resolving power even with lower SNR and snapshot vector than MUSIC technique.*

1. INTRODUCTION

In recent research, DOA estimation is normally considered as a feature of the more conventional field of array processing. A great part of the work in this field, especially in prior days, concentrated on radio direction finding – that is, evaluating the direction of electromagnetic waves receives by one or more antennas. In present scenario, it emerges in numerous engineering applications like wireless communication, radar, radio astronomy, sonar, navigation, rescue and other devices[1].

A signal processing feature of a smart antenna system has focused on the advancement of proficient algorithms for direction of arrival (DOA) estimation.

By using single antenna for DOA estimation, beamwidth of primary lobe get more extensive and appropriate resolution is not acquired. If we attempt to expand the resolution, physical size of antenna is also increased yet it is not a practical approach. To overcome this issue, uniform linear array (ULA) is considered for DOA estimation. ULA is an array comprise of equally spaced antenna of generally same amplitude. Therefore, antenna array is more effective over the single antenna in signal reception and parameter estimation[2].

The numerous algorithms for DOA estimation are Bartlett, Maximum entropy, linear prediction, Capon, Min-norm, MUSIC, ROOT-MUSIC and ESPRIT. Standard methods first evaluate a spatial spectrum and by using local maxima of the spectrum it estimate DOAs. Hence high angular resolution subspace methods such as MUSIC and ROOT-MUSIC algorithms are most widely used[3].ROOT-MUSIC is more effective, as it minimizes the effort by finding the roots of a polynomial rather than plotting the pseudospectrum or examining for highs in the pseudospectrum.

The rest of the paper is sorted as follows. Section2 portrays the system model of ULA. In Section3, subspace based techniques are exhibited. Computer simulation, results and analysis are presented in Section4. Finally, conclusion is given in Section5.

2. SYSTEM MODEL

In general linear array is the *M*-element array. Intended for simplification, we will expect that all elements are similarly separated by the same distance and have level with amplitudes. Afterwards, antenna elements can have random amplitude. Fig. 1 demonstrates *M*-element linear array made up of isotropic radiating antenna elements. It is accepted that the mth element drives the $(m-1)$ element by an electrical phase shift of $\emptyset$ radians.

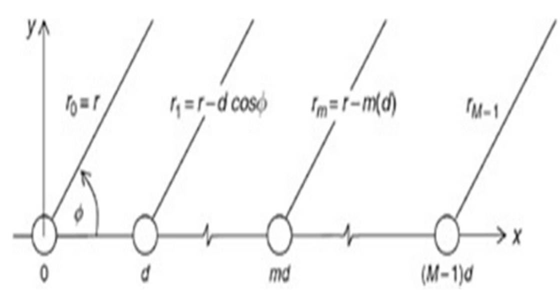

Fig. 1: Uniform linear array

where, number of array elements =*M*,

inter-element spacing= *d*,

number of sources= *D*,

number of snapshots = *K*.

In this model, *D* number of sources $s_1, s_2, \ldots s_D$

are considered with corresponding angles $\emptyset_1, \emptyset_2, \cdots \emptyset_D$, respectively. The incoming signals from sources are represented by their phase and amplitude and their direction of arrival are mention by array propogation vector $\mathbf{b}(\emptyset_i)$. The receive vector is expressed as,

$$\begin{bmatrix} \mathbf{r}_1(k) \\ \mathbf{r}_2(k) \\ \vdots \\ \mathbf{r}_M(k) \end{bmatrix} = [b(\emptyset_1) \quad b(\emptyset_2) \quad \ldots \quad b(\emptyset_D)] \begin{bmatrix} \mathbf{s}_1(k) \\ \mathbf{s}_2(k) \\ \vdots \\ \mathbf{s}_D(k) \end{bmatrix} + \mathbf{W}(k)$$

$$\mathbf{R}(k)=\mathbf{B}(\emptyset)\mathbf{S}(k)+\mathbf{W}(k) \quad (1)$$

where, $\mathbf{R}(k)$ is received matrix with dimension [*MXK*], $\mathbf{S}(k)$ is incoming signal matrix with dimension [*DXK*], $\mathbf{W}(k)$ is additive white Gaussian noise vector with dimension [*MXK*] and $\mathbf{B}(\emptyset)$ is array propogation matirx [4,5] with dimension [*MXD*] of propogation vector $\mathbf{b}(\emptyset_i)$denoted as,

$$b(\emptyset_i) = \begin{bmatrix} 1 \\ e^{j\beta d \sin\emptyset_i} \\ e^{j\beta d2\sin\emptyset_i} \\ \vdots \\ e^{j\beta d(M-1)\sin\emptyset_i} \end{bmatrix}$$

where, β is an incident wave number *(2π/λ) and i* vary from 1,2……*D* .

It is primarily expected that the arriving signals are monochromatic and their quantity is $D< M$. They are time varying in nature. Thus, estimation is based upon time snapshots of the incoming signal.

3. SUBSPACE BASED ALGORITHM

There are two algorithms based upon subspace method are as follows.

3.1 MUSIC algorithm

MUSIC stands for Multiple Signal Classification. This methodology was initially suggests by Schmidt[6] and is a well known high determination eigen structure based strategy. MUSIC provides impartial estimates of the number of signals, the angles of arrival, and the strengths of the waveforms.

Prior to angle estimation, correlation matrix is calculated by the following equation,

$$\mathbf{C}_{rr} = B(\emptyset)\mathbf{C}_{ss}B^H(\emptyset) + \sigma_w^2\mathbf{I} \quad (2)$$

where, $\mathbf{C}_{rr}$ and $\mathbf{C}_{ss}$ represent the autocorrelation of the receiving and incoming signals, respectively. σ_w^2 depicts the variance of noise.

Now compute the eigen values and eigen vectors of C_{rr} and organize the eigen values in ascending order. Eigen vectors which are associated with the smallest (*M-D*) eigen values is the noise subspace ($\mathbf{V}_n$) and remaining D eigen values is the signal subspace ($\mathbf{V}_s$). $\mathbf{V}_n$ and $\mathbf{V}_s$ has matrix dimension *M*X(*M-D*) and *MXD*, respectively. Thus, eigen vectors are subdivided into [$\mathbf{V}_n$,$\mathbf{V}_s$].

$\mathbf{V}_n$ is orthogonal to the array propogation vector

at the direction of arrival $\emptyset_1, \emptyset_2, \ldots\ldots\ldots\ldots, \emptyset_D$.

Due to this orthogonality condition, one can demonstrate that the Euclidean separation $\mathrm{d}^2 = \mathbf{b}^H(\emptyset_i)\mathbf{V}_n\mathbf{V}_n^H\mathbf{b}(\emptyset_i) = 0$

for every single arriving angle $\emptyset_1, \emptyset_2, \ldots, \emptyset_D$. Arranging this distance expression in the denominator generates sharper peaks at the direction of arrival. The MUSIC pseudospectrum is expressed as,

$$G(\emptyset) = \frac{1}{|\mathbf{b}^H(\emptyset_p)\mathbf{V}_n\mathbf{V}_n^H\mathbf{b}(\emptyset_p)|} \quad (3)$$

where, p varies from 1,2,…….M.

3.3 ROOT-MUSIC algorithm

The MUSIC algorithm on the whole can apply to any random array paying little heed to the position of the array elements. Root-MUSIC infers that the MUSIC algorithm is decreased by finding roots of a polynomial rather than just plotting the pseudospectrum or looking for highs in the pseudospectrum. Barabell[7] improved the MUSIC approach for the situation where the receiving antenna is a ULA. Modify the equation (3) by assuming,

$$\mathbf{Q} = \mathbf{V}_n\boldsymbol{V}_n^H \quad (4)$$

Equation (3) can be rewritten as,

$$G(\emptyset) = \frac{1}{|\mathbf{b}^H(\emptyset_p)\mathbf{Q}\mathbf{b}(\emptyset_p)|} \quad (5)$$

If receiving element is a ULA, the p^{th} element of the array propogation vector is stated by

$$\mathbf{b}_p(\emptyset) = e^{j\beta d(p-1)sin\emptyset}$$

where, p=1,2,3,……..,M

The denominator of equation (5) can be evaluated as

$$\mathbf{b}^H(\emptyset_p)\mathbf{Q}\mathbf{b}(\emptyset_p) = \sum_{v=-M+1}^{M-1} q_v e^{j\beta dv Sin(\emptyset)} \quad (6)$$

where, q_v represents the addition of the diagonal elements of Q along the v^{th} diagonal illustrated as,

$$q_v = \sum_{n-p=v} Q_{pn} \quad (7)$$

The value of *v* corresponds to –*M*+1,.,0,1,…*M*-1

Now equation (6) can be represented as a polynomial in z-domain as,

$y(z) = \sum_{v=-M+1}^{M-1} q_v z^v$ where, $z = e^{j\beta d \sin(\emptyset)}$(8)

The degree of polynomial is 2(*M*-1).

Now evaluate the roots of *y*(*z*) and plot all these roots in z-domain.

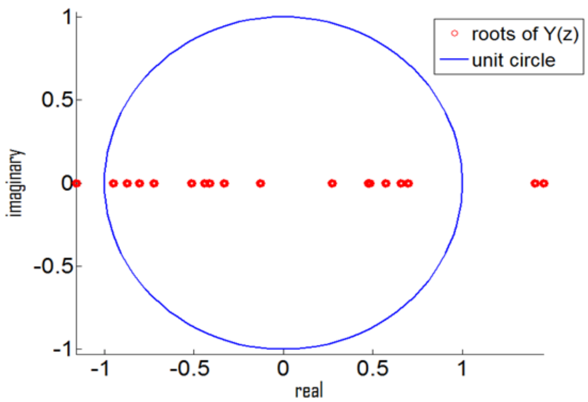

Fig. 2. 2(M-1) Roots of Polynomial y(z)

Now determine the *M*-1 roots of the a polynomial that are nearest and inside the unit circle as shown in fig. 3

The roots which lie nearest to the unity circle indicate the poles of MUSIC pseudospectrum and approach is termed as root-MUSIC algorithm.

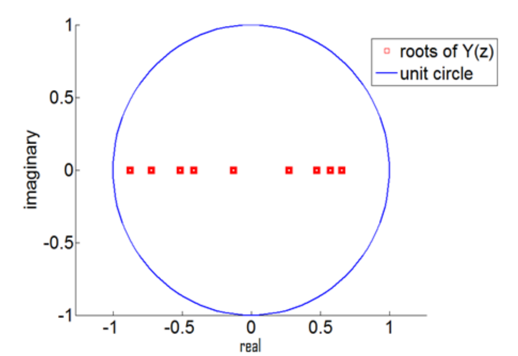

Fig. 3: (M-1) Roots of Polynomial y(z)

Compute the direction of arrival by the following equation,

$$\emptyset_e = -\sin^{-1}\frac{1}{\beta d}\arg(z_e) \quad (9)$$

where, e=1,2,……,2(M-1)

4. SIMULATIONS, RESULTS AND ANALYSIS

The impact of changing distinctive parameters on the execution of the MUSIC and ROOT-MUSIC algorithm has been examined. For the sake of simplicity, ULA is considered with fixed number of sources.

In these simulations, number of sources are 4 with the corresponding angles 20°, 40°, 50° and 60°, respectively. The value of signal to noise ratio is 1 dB. Now, following parameters are varied that is number of snapshots(*K*) and element spacing(*d*).

For both the cases simulation results are compared with fig. 1 and the following parameters are *K*=1000 and *d*=0.5*λ*.

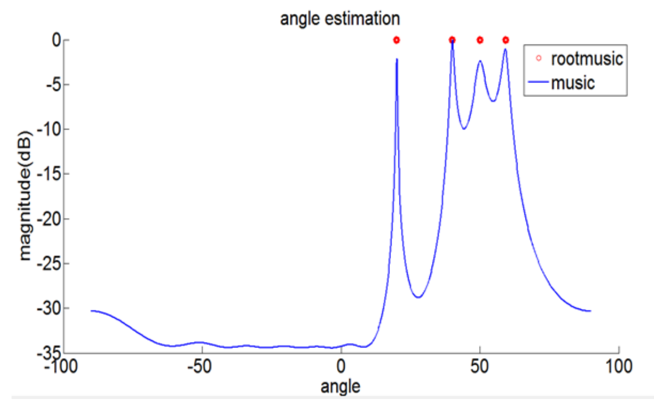

Fig. 4 Angle Estimation of ROOT-MUSIC and MUSIC Algorithm, respectively for K=1000 and d=0.5λ at 1dB.

Table 1: Comparison of Angle Estimation of ROOT-MUSIC and MUSIC Algorithm, respectively for K=1000 and d=0.5λ at 1dB.

K=1000 and d=0.5λ				
Given angle (ø)	Estimated		Mean Square Error(%)	
	MUSIC	ROOT-MUSIC	MUSIC	ROOT-MUSIC
20°	20.09°	20.0128°	0.45	0.064
40°	40.27°	40.0335°	0.675	0.08375
50°	50.18°	49.9730°	0.36	0.054
60°	59.01°	59.3222°	1.65	1.13

From Fig. 4 and table 1, we observe that the estimation of DOA is accurate for both algorithms.

4.1 Varying number of snapshots

Fig.4 and fig.5 measure the difference between the angle estimation of MUSIC and ROOT-MUSIC algorithm for number of snapshots *K*=1000 and 100, respectively with *d*=0.5λ.

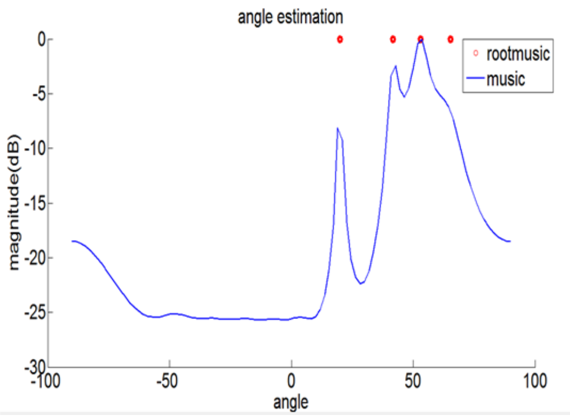

Fig. 5: Angle Estimation of ROOT-MUSIC and MUSIC Algorithm, respectively for K=100 and d=0.5λ at 1dB.

Table 2: Comparison of Angle Estimation of ROOT-MUSIC and MUSIC Algorithm, respectively for K=100 and d=0.5λ at 1dB.

K=100 and d=0.5λ				
Given angle (ø)	Estimated		Mean Square Error(%)	
	MUSIC	ROOT-MUSIC	MUSIC	ROOT-MUSIC
20°	19.09°	19.90°	4.55	0.5
40°	42.73°	40.61°	6.825	1.525
50°	53.64°	50.9455°	6.48	1.891
60°	-	60.4615°	-	0.769

Fig. 5 and Table 2, illustrates that by decreasing the number of snapshots K, all angles are estimated in ROOT-MUSIC algorithm but one angle is not estimated in MUSIC algorithm.

4.2 Varying the element spacing

Fig.4 and fig.6 measure the difference between the angle estimation of MUSIC and ROOT-MUSIC algorithm for element spacing d=0.5λ and 0.4λ, respectively with K=1000.

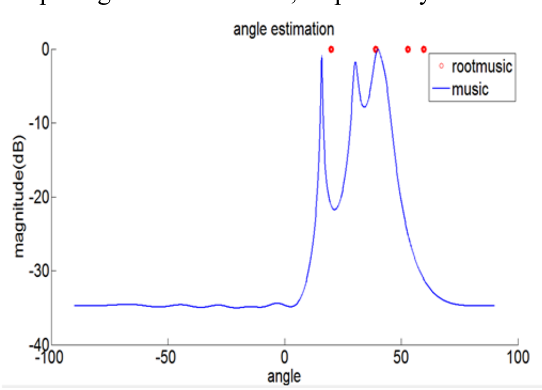

Fig. 6. Angle Estimation of ROOT-MUSIC and MUSIC Algorithm, respectively for K=1000 and d=0.4λ at 1dB.

Table 3. Comparison of Angle Estimation of ROOT- MUSIC and MUSIC Algorithm, respectively for *K*=1000 and *d*=0.4λ *at* 1dB.

K=1000 and d=0.4λ				
Given angle (ø)	Estimated		Mean Square Error(%)	
	MUSIC	ROOT-MUSIC	MUSIC	ROOT-MUSIC
20°	15.95°	19.9858°	20.25	0.071
40°	30.36°	38.9974°	24.1	2.5065
50°	39.55°	52.9246°	20.9	5.84
60°	-	59.7345°	-	0.4425

Fig. 6 and Table3, outlines that by reducing the element spacing d, the mean square error(MSE) of MUSIC algorithm has rapidly increased as compare to MSE of ROOT- MUSIC algorithm.

It is observed that the MUSIC algorithm is not capable but rather at times the outcomes acquired with this algorithm are adequate like K=1000 and d=0.5λ, while ROOT-MUSIC perform well in all the above conditions.

5. CONCLUSION

In this paper, the comparison of DOA estimation is done by MUSIC and ROOT-MUSIC algorithm based upon subspace technique by varying the parameters of antenna. Simulation results exhibits that on lessening the number of snapshot vectors and element spacing, some crests of pseudo spectra is disappeared in MUSIC algorithm. Hence MUSIC indicates more MSE and the precision of ROOT MUSIC is far superior to the MUSIC algorithm.

REFERENCES

[1]E. Tuncer and B. Friedlander, “Classical and Modern Direction-of-Arrival Estimation,” Ed. Elsevier,USA, 2009, ch. 1.

[2] H. Hwang et al., “Direction of Arrival Estimation using a Root-MUSIC Algorithm,” *Proceedings of the International MultiConference of Engineers and Computer Scientists,* vol. II,Hong Kong, March 2008.

[3] Takao K. and Kikuma N., 1987, “An adaptive array utilizing an adaptive spatial averaging technique for multipath environments”, IEEE Trans. AP, m, 1389-1 396.

[4] Litva, J., and T.K-Y. Lo, *Digital Beamforming in Wireless Communications*, Artech House, 1996.

[5]. Monzingo, R.A., and T.W. Miller, *Introduction to Adaptive Arrays*, Wiley, New York,1980.

[6] Schmidt, R., “Multiple Emitter Location and Signal Parameter Estimation,” *IEEETransactions on Antenna. Propogation.*, Vol. AP-34, No. 2, pp. 276–280, March 1986.

[7] Barabell, A., “Improving the Resolution of Eigenstructure-Based Direction-Finding Algorithms,” *Proceedings of ICASSP*, Boston, MA, pp. 336–339, 1983.

[8] Mahmoud Mohanna, Mohamed L. Rabeh, Emad M. Zieur, and Sherif Hekala, “Optimization of MUSIC algorithm for angle of arrival estimation in wireless communications, ” *NRIAG Journal of Astronomy and Geophysics (2013)* 2, 116–124.